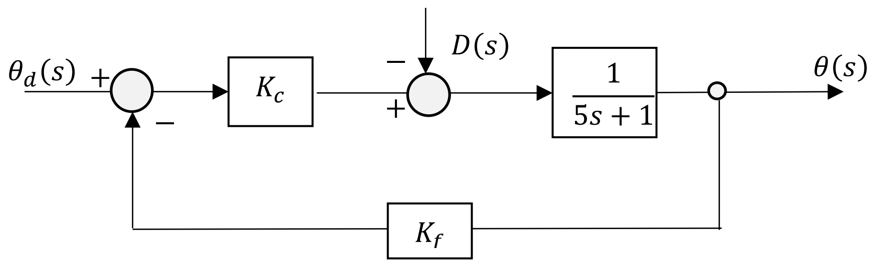
Control – open book examination

Figure a:



a) Show that the system presented as a block diagram in figure (a) has the transfer function

$$G(s) = \frac{\theta(s)}{\theta_d(s)} = \frac{K_c}{5s + 1 + K_c K_f}$$

Where K_c is the controller gain, K_f is the feedback gain, and $D(s)$ is a disturbance signal. [3 marks]

Solution:

Step 1	ignore D(s) as it's irrelevant to the question. This gives a forward transfer function of $\frac{K_c}{5s+1}$.	0.5 marks
Step 2	$\theta(s) = \left(\theta_d(s) - K_f\,\theta(s)\right)\left(\frac{K_c}{5s+1}\right)$ $\theta(s)(5s+1) = K_c\theta_d(s) - K_cK_f\,\theta(s)$ $\theta(s)\left(5s+1+K_cK_f\right) = K_c\theta_d(s)$	1 mark 1 mark
Step 3	Present in the correct form as the transfer function: $G(s) = \frac{\theta(s)}{\theta_d(s)} = \frac{K_c}{5s+1+K_cK_f}$	0.5 marks

b) If the input to the system in part (a) is a step input $\theta(s) = \frac{1}{s}$, use the final value theorem to calculate the steady state error for $K_c = 1$, $K_f = 1$, and $D(s) = 0$. Show your working.

[3 marks]

Solution:

Step 1	Put in the values from the question to give: $\theta(s) = \frac{1}{s}\left(\frac{1}{5s+2}\right)$	1 mark
Step 2	Error is given by $\theta_d(s) - \theta(s) = \frac{1}{s} - \frac{1}{s}\left(\frac{1}{5s+2}\right) = \frac{1}{s}\left(\frac{5s+2}{5s+2} - \frac{1}{5s+2}\right) = \frac{1}{s}\left(\frac{5s+1}{5s+2}\right)$	1 mark
Step 3	Use the final value theorem to give the error: $\lim_{t\to\infty}(\theta_d - \theta) = \lim_{s\to 0} s\left(\theta_d(s) - \theta(s)\right) = \frac{s}{s}\left(\frac{5s+1}{5s+2}\right) = \frac{1}{2}\ or\ 0.5$	1 mark

Alternative time domain solution:

Step 1	Put in the values from the question to give: $$\theta(s) = \frac{1}{s}\left(\frac{1}{5s+2}\right)$$	1 mark
Step 2	Using the table of Laplace Transforms: $$1 - e^{-at} \xrightarrow{\mathcal{L}} \frac{a}{s(s+a)}$$ $$\theta(s) = \frac{1}{s}\left(\frac{1}{5s+2}\right) = \frac{1}{2}\left(\frac{1}{s}\left(\frac{0.4}{s+0.4}\right)\right)$$	1 mark
Step 3	Inverse Laplace transform gives: $$\theta(t) = \frac{1}{2}(1 - e^{-0.4t})$$ And so $\lim_{t\to\infty}(\theta_d - \theta) = 1 - \frac{1}{2}(1 - e^{-\infty}) = \frac{1}{2}$ *or* 0.5	0.5 marks 0.5 marks

The question explicitly asks for the final value theorem: deduct 1 mark if the correct answer is gained using the time domain method.

c) Show that the rise time to reach 90% of the final steady state value for the step input in part (b) is 5.76s.

[3 marks]

Solution:

Step 1	Students should already have: $$\theta(s) = \frac{1}{s}\left(\frac{1}{5s+2}\right)$$	
Step 2	Solve in the time domain using the table of Laplace Transforms: $$1 - e^{-at} \xrightarrow{\mathcal{L}} \frac{a}{s(s+a)}$$ $$\theta(s) = \frac{1}{s}\left(\frac{1}{5s+2}\right) = \frac{1}{2}\left(\frac{1}{s}\left(\frac{0.4}{s+0.4}\right)\right)$$ Inverse Laplace transform gives: $$\theta(t) = \frac{1}{2}(1 - e^{-0.4t})$$	1 mark 1 mark
Step 3	Inverse Laplace transform gives: If $1 - e^{-0.4t} = 0.9$ then $e^{-0.4t} = 0.1$ $$0.4t = -\ln 0.1 = 2.30$$ $$t = 5.76s$$	1 mark

d) With the Routh-Hurwitz criteria, show that a system with the characteristic equation

$$s^4 + 2s^3 + 3s^2 + 4s + 5$$

will be unstable.

[3 marks]

Solution:

<table>
<tr><td>Step 1</td><td>Criterion 1: No change of sign in the characteristic equation so this criterion is satisfied.</td><td>0.5 marks</td></tr>
<tr><td>Step 2</td><td>Begin the Routh table by filling in the first two lines correctly:
<table>
<tr><td>s^4</td><td>1</td><td>3</td><td>5</td><td>0</td></tr>
<tr><td>s^3</td><td>2</td><td>4</td><td>0</td><td>0</td></tr>
<tr><td>s^2</td><td></td><td></td><td></td><td></td></tr>
<tr><td>s</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td></tr>
</table></td><td>0.5 marks</td></tr>
<tr><td>Step 3</td><td>Calculate the missing values to give the full Routh Array.:
<table>
<tr><td>s^4</td><td>1</td><td>3</td><td>5</td><td>0</td></tr>
<tr><td>s^3</td><td>2</td><td>4</td><td>0</td><td>0</td></tr>
<tr><td>s^2</td><td>1</td><td>5</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>-6</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>5</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td>1 mark</td></tr>
<tr><td>Step 4</td><td>Comment on the first column of the Routh Array:
There are two changes of sign in the column and so we can deduce that there are two roots on the right hand side of the s plane, i.e. with a positive real value of σ for $s = \sigma + i\omega$. This means that the magnitude of the response will increase exponentially over time and hence the system will be unstable.</td><td>1 mark for the essential red bits of the full formal expansion here</td></tr>
</table>